B.Sc. Part–III (Semester–V) Examination

MATHEMATICS

MATHEMATICAL ANALYSIS

Paper–IX

Time : Three Hours] [Maximum Marks : 60

Note : — (1) Question No. **1** is compulsory, attempt it once only.

(2) Attempt **one** question from each unit.

1. Choose the correct alternative : 10

(i) Consider P = (1, 2, 3, 4) is a partition of interval [1, 4], then $\mu(P)$ is ....

(a) 1 (b) 2

(c) 3 (d) 4

(ii) Let f be a bounded function defined on [a, b] and P be any partition of [a, b], P* be refinement of P. Then L(P,f) and L(P*,f) satisfy.........

(a) $L(P, f) \leq L(P^*, f)$ (b) $L(P, f) \geq U(P^*, f)$

(c) $L(P, f) \geq L(P^*, f)$ (d) None of these

(iii) An integral $\int_0^\infty e^{-kx}x^{n-1}dx$ is :

(a) $k^n \sqrt{n}$ (b) $\frac{\sqrt{n}}{k^n}$

(c) k^n (d) $\sqrt{n}$

(iv) The value of $\sqrt{\frac{1}{2}}$ is ....

(a) $\frac{1}{2}$ (b) 1

(c) $\sqrt{\pi}$ (d) π

(v) In polar form, the Cauchy Riemann equation can be ________.

(a) $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = \frac{-1}{r}\frac{\partial u}{\partial \theta}$

(b) $\frac{\partial u}{\partial r} = -\frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = \frac{1}{r}\frac{\partial u}{\partial \theta}$

(c) $\frac{\partial u}{\partial r} = -\frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$

(d) None of these

(vi) If w = u + iv is analytic function in D, then $\frac{dw}{dz}$ is :

(a) $-\frac{\partial w}{\partial x}$ (b) $\frac{\partial w}{\partial y}$

(c) $-\frac{\partial w}{\partial y}$ (d) $\frac{\partial w}{\partial x}$

(vii) A Mobius transformation w = az, a is real number, is :

(a) Rotation transformation (b) Magnification transformation

(c) Translation transformation (d) None of these

(viii) A bilinear transformation with only one fixed point is :

(a) Loxodromic (b) Parabolic

(c) Elliptic (d) Hyperbolic

(ix) For any collection of $\{A_\alpha\}$ open sets, $\underset{\alpha}{\cup} A_\alpha$ is :

(a) Closed (b) Open

(c) Semi-open (d) None of these

(x) Let X denote a discrete metric space and $A \subseteq X$ then :

(a) A is open

(b) A is closed

(c) A is both open and closed iff $A = \phi$ and $A = X$

(d) A is both open and closed

UNIT—I

2. (a) Let the functions f and g be integrable on [a, b] and let α be any real constant then 5

(i) $\alpha \in R[a, b]$ and $\int_a^b \alpha\, dx = \alpha(b-a)$

(ii) $\alpha f \in \mathrm{R}\,[a,\ b]$ and $\int_a^b (\alpha f)(x)\,dx = \alpha \int_a^b f\ dx.$

(b) Refinement of a partition P increases lower sums and decreases upper sums 5

i.e. $\mathrm{L}(P, f) \le \mathrm{L}(P^*, f)$ and $\mathrm{U}(P, f) \ge \mathrm{U}(P^*, f)$

OR

3. (c) If f is a bounded and integrable function over [a, b] and M, m are the bounds of f over [a, b], then prove that 5

$$m(b-a) \le \int_a^b f(x)\,dx \le \mathrm{M}(b-a)$$

(d) If $f \in R\ [a,\ b]$, then $\mathrm{F}:[a, b] \to \mathrm{R}$ defined by $F(x) = \int_a^x f(t)$ dt is continuous on [a, b]. If f is continuous at $x_0 \in [a,\ b]$ then F is differentiable at x_0 with $\mathrm{F}'(x_0) = f(x_0)$ and if f is continuous on [a, b] then F is differentiable on [a, b] with $\mathrm{F}'(x) = f(x)\ \forall x\ \in [a,b]$. 5

UNIT—II

4. (a) Let $f(x), g(x) \in C.\ a \le x < \infty$ and $0 \le f(x) \le g(x), \forall x \ge a$ then 5

(i) $\int_a^\infty g(x)\,dx$ converges $\Rightarrow \int_a^\infty f(x)\,dx$ converges

(ii) $\int_a^\infty f(x)\,dx = \infty\ \Rightarrow \int_a^\infty g(x)\,dx = \infty$

(b) Prove that $\int_1^\infty e^{-x} x^{n-1}\,dx$ is convergent for any n. 5

OR

5. (c) Show that the integrals :

(i) $\int_0^\infty \frac{\cos x}{\sqrt{1+x^3}}\,dx$ and

(ii) $\int_1^\infty \frac{x\,dx}{3x^4 + 5x^2 + 1}\,dx$

Converges absolutely. 5

(d) Prove that $\Gamma(2m) = \dfrac{2^{2m-1}}{\sqrt{\pi}}\Gamma(m)\,\Gamma\!\left(m+\dfrac{1}{2}\right)$ 5

UNIT—III

6. (a) Prove that necessary condition that $f(z) = u(x, y) + i\, v(x, y)$ be analytic in a region D is that $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}, \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$ in D. 5

(b) Show that $u = y^3 - 3x^2 y$ is harmonic and find its harmonic conjugate function. Hence find the analytic function $f(z) = u + iv$. 5

OR

7. (c) If f(z) is analytic function with constant modules show that f(z) is constant. 5

(d) If $w = \phi + i\Psi$ represents the complex potential for an electric field and $\Psi = (x^2 - y^2) + \dfrac{x}{x^2 + y^2}$ · Determine the function ϕ. 5

UNIT—IV

8. (a) Find the fixed points of the transformation $w = \dfrac{z-1}{z+1}$. State whether it is hyperbolic, elliptic or loxodromic. 5

(b) Prove that, every bilinear transformation with two non-infinite fixed points α, β is of the form $\dfrac{w-\alpha}{w-\beta} = k\dfrac{z-\alpha}{z-\beta}$, where k is the constant. 5

OR

9. (c) Under the transformation $w = \dfrac{1}{z}$ a straight line L in the z-plane is mapped into : 5

(i) a circle if does not pass through the origin $z = 0$.

(ii) a straight line if L passes through the origin $z = 0$.

(d) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -1$. Find the image of $|z| < 1$. 5

UNIT—V

10. (a) Let X be a non-empty set and let d be a real function of ordered pairs of elements of X which satisfies the following conditions. 5

(i) $d(x, y) = 0 \Leftrightarrow x = y$

(ii) $d(x, y) \le d(x, z) + d(y, z)$ show that d is a metric on X.

(b) Let $\{A_\alpha\}$ be a finite or infinite collection of sets A_α then $\left[\bigcup_\alpha A_\alpha\right]^c = \bigcap_\alpha A_\alpha^c$ 5

OR

11. (c) Let (X, d) be a metric space and $A \subseteq X$. Prove that A is closed iff A contains its boundary b (A) $\subseteq$ A. 5

(d) Let (X, d) be a metric space and $x, y, x', y' \in X$ show that 5

$$|d(x,y) - d(x',y')| \le d(x,x') + d(y, y')$$